

# $|V_{cb}|$ and $|V_{ub}|$ : Theoretical Developments<sup>1</sup>

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The determinations of  $|V_{cb}|$  and  $|V_{ub}|$  from semileptonic  $B$  decays are reviewed with emphasis on recent developments and theoretical uncertainties. Future prospects and limitations are also discussed.  
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## 1 Introduction

Since  $\sin 2\beta$ , the  $CP$  asymmetry in  $B \rightarrow \psi K_S$  and related modes [1], is consistent with  $\epsilon_K$ ,  $|V_{ub}|$ , and  $B_{d,s}$  mixing, searching for new flavor physics will require a combination of “redundant” and precise measurements that relate in the Standard Model (SM) to the same CKM elements. The determination of  $|V_{cb}|$  is important because the uncertainties in  $\epsilon_K$  and in  $K \rightarrow \pi \nu \bar{\nu}$  are proportional to  $|V_{cb}|^4$ , while the uncertainty of  $|V_{ub}|$  dominates the error of a side of the unitarity triangle. Some of the best strategies to look for new physics include comparing angles and sides of the unitarity triangle, and results from tree and loop processes, and semileptonic decays are crucial for this. Processes mediated by flavor-changing neutral currents, such as  $b \rightarrow q \gamma$ ,  $b \rightarrow q \ell^+ \ell^-$ , and  $b \rightarrow q \nu \bar{\nu}$  ( $q = s, d$ ) are sensitive probes of the SM, and the theoretical tools to analyze these are the same as for  $|V_{xb}|$ . It is the accuracy of the theory that ultimately limits the sensitivity to new physics [2].

To illustrate where the future might take us, Fig. 1 shows CKM fits assuming that  $\sin 2\beta$  equals its present central value with half the error, and that  $|V_{ub}|$  is about  $1.5\sigma$  lower [higher] than its central value with 5% experimental and theoretical errors:  $|V_{ub}| = (3.0 \pm 0.15 \pm 0.15) \times 10^{-3}$  [ $(5.0 \pm 0.25 \pm 0.25) \times 10^{-3}$ ]. These fits are motivated by the fact that recent exclusive [inclusive] measurements of  $|V_{ub}|$  appear to be on the low [high] side [3]. The resulting central values of the angle  $\gamma$  differ by  $25^\circ$ , and the SM value of  $\Delta m_s$  is near the minimum [maximum] of its presently allowed range. Clearly, an accuracy of  $\sigma(|V_{ub}|) \sim 5\%$  is very desirable.

To believe at some point in the future that a discrepancy between measurements is due to new physics, model independent predictions are crucial. Results that depend on modeling nonperturbative strong interaction effects cannot prove that there is new flavor physics beyond the SM. Model independent predictions are those where the theoretical uncertainties are suppressed by powers of small parameters, typically  $\Lambda_{\text{QCD}}/m_b$ ,  $m_s/\Lambda_{\chi\text{SB}}$ ,  $\alpha_s(m_b)$ , etc. Still, in most cases, there are uncertainties at some order, which cannot be estimated model independently. If the goal is to test the Standard Model, one must assign sizable uncertainties to such “small corrections” not known from first principles.

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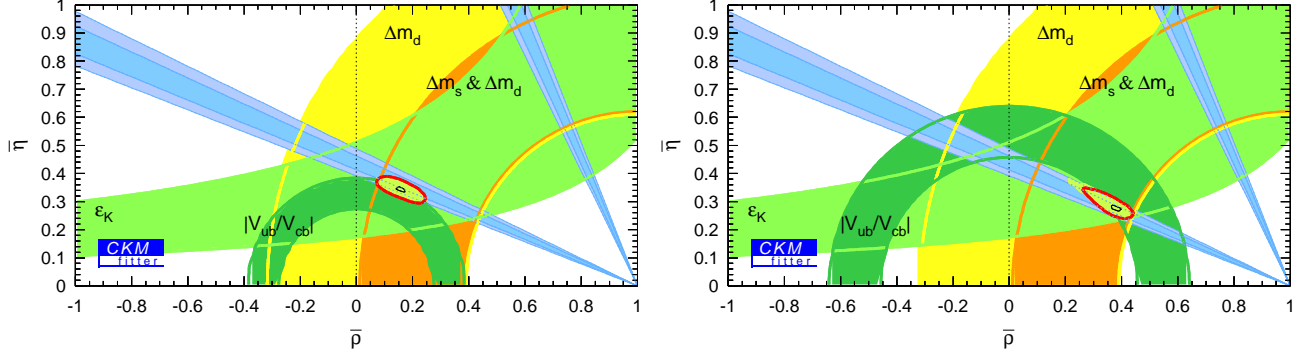


Figure 1: CKM fits with hypothetical values,  $|V_{ub}| = (3.0 \pm 0.15 \pm 0.15) \times 10^{-3}$  (left) and  $(5.0 \pm 0.25 \pm 0.25) \times 10^{-3}$  (right), and  $\sin 2\beta =$  its present world average with half the error [4].

Over the last decade most of the theoretical progress in understanding  $B$  decays utilized that  $m_b$  is much larger than  $\Lambda_{\text{QCD}}$ . However, depending on the process under consideration, the relevant hadronic scale may or may not be numerically much smaller than  $m_b$  (and, especially,  $m_c$ ). For example,  $f_\pi$ ,  $m_\rho$ , and  $m_K^2/m_s$  are all of order  $\Lambda_{\text{QCD}}$ , but their numerical values span an order of magnitude. In most cases experimental guidance is needed to decide how well the theory works for various processes.

## 2 $B \rightarrow X_c \ell \bar{\nu}$

### 2.1 $|V_{cb}|$ exclusive: Heavy Quark Symmetry

In mesons composed of a heavy quark and a light antiquark (plus gluons and  $q\bar{q}$  pairs), the energy scale of strong interactions is small compared to the heavy quark mass. The heavy quark acts as a static point-like color source with fixed four-velocity, which cannot be altered by the soft gluons responsible for confinement. Thus, the configuration of the light degrees of freedom (“brown muck”) become insensitive to the spin and flavor (mass) of the heavy quark.

Heavy quark symmetry (HQS) [5] is especially predictive for  $B \rightarrow D^{(*)}$  semileptonic decays. When the weak current changes suddenly (on a time scale  $\ll \Lambda_{\text{QCD}}^{-1}$ ) the flavor  $b \rightarrow c$ , the momentum  $\vec{p}_b \rightarrow \vec{p}_c$ , and possibly the spin,  $\vec{s}_b \rightarrow \vec{s}_c$ , the brown muck only feels that the four-velocity of the static color source in the center of the meson changed,  $v_b \rightarrow v_c$ . Therefore, the form factors that describe the wave function overlap between the initial and final mesons become independent of the Dirac structure of weak current, and can only depend on a scalar quantity,  $w \equiv v_b \cdot v_c$ . Thus all form factors are related to a single Isgur-Wise function,  $\xi(v_b \cdot v_c)$ , which contains all the low energy nonperturbative hadronic physics relevant for these decays. Moreover,  $\xi(1) = 1$ , because at “zero recoil”,  $w = 1$ , where the  $c$  quark is at rest in the  $b$  quark’s rest frame, the configuration of the brown muck does not change at all.

The determination of  $|V_{cb}|$  from exclusive  $B \rightarrow D^{(*)} \ell \bar{\nu}$  decay uses an extrapolation of the measured rate to zero recoil,  $w = 1$ . The rates can be schematically written as

$$\frac{d\Gamma(B \rightarrow D^{(*)} \ell \bar{\nu})}{dw} = (\text{known factors}) |V_{cb}|^2 \begin{cases} (w^2 - 1)^{1/2} \mathcal{F}_*^2(w), & \text{for } B \rightarrow D^*, \\ (w^2 - 1)^{3/2} \mathcal{F}^2(w), & \text{for } B \rightarrow D. \end{cases} \quad (1)$$

In the heavy quark limit  $\mathcal{F}(w) = \mathcal{F}_*(w) = \xi(w)$ , and in particular  $\mathcal{F}_*(1) = 1$ , allowing for a model independent determination of  $|V_{cb}|$ . The corrections to the  $m_Q \rightarrow \infty$  limit ( $Q = b, c$ ) can be organized in a simultaneous expansion in  $\alpha_s(m_Q)$  and  $\Lambda_{\text{QCD}}/m_Q$ , of the form

$$\begin{aligned}\mathcal{F}_*(1) &= 1_{\text{(Isgur-Wise)}} + c_A(\alpha_s) + \frac{0_{\text{(Luke)}}}{m_Q} + \frac{\text{(lattice or models)}}{m_Q^2} + \dots, \\ \mathcal{F}(1) &= 1_{\text{(Isgur-Wise)}} + c_V(\alpha_s) + \frac{\text{(lattice or models)}}{m_Q} + \dots.\end{aligned}\quad (2)$$

The perturbative corrections  $c_A = -0.04$  and  $c_V = 0.02$  are known to order  $\alpha_s^2$  [6], and higher order terms should be below the 1% level. The order  $\Lambda_{\text{QCD}}/m_Q$  correction to  $\mathcal{F}_*(1)$  vanishes due to Luke's theorem [7]. The terms indicated by (lattice or models) are only known using phenomenological models or quenched lattice QCD at present. This is why the determination of  $|V_{cb}|$  is more reliable from  $B \rightarrow D^* \ell \bar{\nu}$  than from  $B \rightarrow D \ell \bar{\nu}$ , although both QCD sum rules [8] and quenched lattice QCD [9] suggest that the  $\Lambda_{\text{QCD}}/m_{b,c}$  correction to  $\mathcal{F}(1)$  is small (giving  $\mathcal{F}(1) = 1.02 \pm 0.08$  and  $1.06 \pm 0.02$ , respectively). The rate near  $w = 1$  is larger in  $B \rightarrow D^* \ell \bar{\nu}$  than in  $B \rightarrow D \ell \bar{\nu}$ , because of the  $w^2 - 1$  helicity suppression of the latter, yielding [10]

$$|V_{cb}| \mathcal{F}_*(1) = (36.7 \pm 0.8) \times 10^{-3}, \quad |V_{cb}| \mathcal{F}(1) = (42.1 \pm 3.7) \times 10^{-3}. \quad (3)$$

Using  $\mathcal{F}_*(1) = 0.91 \pm 0.04$  (an estimate unchanged for many years [11] and supported by a recent quenched lattice calculation [12]), yields  $|V_{cb}| = (40.2 \pm 0.9_{\text{exp}} \pm 1.8_{\text{th}}) \times 10^{-3}$ . The  $B \rightarrow D \ell \bar{\nu}$  data is consistent, but to make a real test (and to further reduce the theoretical error of  $|V_{cb}|$ ), unquenched lattice calculations of  $\mathcal{F}_*(1)$  are needed.

Another important theoretical input is the shape of  $\mathcal{F}_*(w)$  used for fitting the data. Expanding about zero recoil, one writes  $\mathcal{F}_*(w) = \mathcal{F}_*(1) [1 - \rho_{(*)}^2(w - 1) + c_{(*)}(w - 1)^2 + \dots]$ . Knowing the slope,  $\rho_{(*)}^2$ , is important because it has a large correlation with the extracted value of  $|V_{cb}| \mathcal{F}_*(1)$ . Analyticity imposes stringent constraints between  $\rho_{(*)}^2$  and the curvature,  $c_{(*)}$  [13], which is used in the fits to obtain Eq. (3). The  $B \rightarrow D \ell \bar{\nu}$  measurement is also important, because HQS constrains the differences  $\rho_*^2 - \rho^2$  and  $c_* - c$  [14], and computing  $\mathcal{F}(1)$  on the lattice is not harder than  $\mathcal{F}_*(1)$ . Sum rules have also been used to constrain  $\mathcal{F}_*(1)$  and the slope parameter [15], and very recently a new set of recursive bounds on all derivatives of the Isgur-Wise function at zero recoil were obtained [16]

$$(-1)^n \xi^{(n)}(1) \geq \frac{2n+1}{4} [(-1)^{n-1} \xi^{(n-1)}(1)] \quad \Rightarrow \quad (-1)^n \xi^{(n)}(1) \geq \frac{(2n+1)!!}{2^{2n}}. \quad (4)$$

An important ingredient in the sum rules are the excited states' contributions, so their precise understanding will improve the determination of  $|V_{cb}|$  from both exclusive and inclusive decays. How to best use all the information on the shapes (of both the  $B \rightarrow D^* \ell \bar{\nu}$  form factors and the  $w$ -spectra) still appears a somewhat open question where progress could be made.

## 2.2 $|V_{cb}|$ inclusive: OPE and HQET parameters

In the large  $m_b$  limit, there is a simple argument based on a separation of scales that the inclusive rate may be modeled by the decay of a free  $b$  quark. The  $b$  quark decay mediated by

the weak interaction takes place on a time scale that is much shorter than the time it takes the quarks in the final state to hadronize. Once the  $b$  quark has decayed, the probability that the decay products will hadronize somehow is unity, and we need not know the (uncalculable) probabilities of hadronization to specific final states.

The above argument can be made precise using an operator product expansion (OPE). When the energy release to the final hadronic state is large, the forward scattering amplitude (whose imaginary part gives the decay rate) can be expanded in local operators,

$$p_b = m_b v + k \quad p = m_b v - q + k \quad = \quad \text{[Tree-level diagram]} + \frac{0}{m_b} \text{[Diagram]} + \frac{1}{m_b^2} \text{[Diagram]} + \dots \quad (5)$$

The OPE allows the model independent computation of inclusive semileptonic  $B$  decay rates in a series in  $\Lambda_{\text{QCD}}/m_b$  and  $\alpha_s(m_b)$  [17], schematically as

$$d\Gamma = \left( \begin{array}{c} b \text{ quark} \\ \text{decay} \end{array} \right) \times \left[ 1 + \frac{0}{m_b} + \frac{f(\lambda_1, \lambda_2)}{m_b^2} + \dots + \alpha_s(\dots) + \alpha_s^2(\dots) + \dots \right]. \quad (6)$$

It proves that semileptonic rates in the  $m_b \rightarrow \infty$  limit are given by  $b$  quark decay, and the leading nonperturbative corrections suppressed by  $\Lambda_{\text{QCD}}^2/m_b^2$  can be parameterized by two HQET matrix elements,  $\lambda_1$  and  $\lambda_2$ . Most quantities of interest have been computed including the order  $\Lambda_{\text{QCD}}^3/m_b^3$  nonperturbative corrections (which are parameterized by six more hadronic matrix elements,  $\rho_{1,2}$  and  $\tau_{1,\dots,4}$ ), while the perturbation series at leading order in  $\Lambda_{\text{QCD}}/m_b$  are known including the  $\alpha_s$  and  $\alpha_s^2 \beta_0$  terms ( $\beta_0 = 11 - 2n_f/3$  is the first coefficient of the QCD  $\beta$ -function, and this term often dominates at order  $\alpha_s^2$ ).

For fully inclusive quantities, such as the  $B \rightarrow X_c \ell \bar{\nu}$  rate, the OPE calculation should be under good control. The theoretical uncertainties in this case come from the error in a short distance  $b$  quark mass (whatever way it is defined), the perturbation series, and the nonperturbative corrections. There has been a lot of recent progress in determining a correlated range of  $|V_{cb}|$ ,  $m_b$ ,  $\lambda_1$  and the HQET matrix elements at order  $\Lambda_{\text{QCD}}^3/m_b^3$  from measurements of shape variables. The idea is to compare the OPE predictions with data for shapes of decay distributions (spectral moments) that are independent of CKM elements. Since different spectra have different dependence on  $m_b$ ,  $\lambda_1$ , etc., a simultaneous fit to many moments and to the semileptonic width allows both the determination of the hadronic parameters and  $|V_{cb}|$ , and tests the validity of the whole approach. The observables which received the most attention are the charged lepton energy [18, 19] and hadronic invariant mass [20, 19] spectra in  $B \rightarrow X_c \ell \bar{\nu}$ , and the photon energy spectrum in  $B \rightarrow X_s \gamma$  [21]. Their measurements show improving consistency, and were discussed elsewhere at this conference [22].

Two recent global fits used somewhat different, but in principle equivalent, approaches. In Ref. [23] both the  $B^* - B$  and  $D^* - D$  mass differences are used to constrain linear combinations of  $\lambda_2$  and some of the  $\Lambda_{\text{QCD}}^3/m_b^3$  matrix elements, and the fit contained seven unknowns:  $|V_{cb}|$ ,  $m_b$ ,  $\lambda_1$ ,  $\rho_1$ ,  $\tau_1 - 3\tau_4$ ,  $\tau_2 + \tau_4$ ,  $\tau_3 + 3\tau_4$ . In Ref. [24] no expansion in  $m_c$  is performed, and

in this case the seven free parameters are:  $|V_{cb}|$ ,  $m_b$ ,  $m_c$ ,  $\lambda_1$ ,  $\lambda_2$ ,  $\rho_1$ ,  $\rho_2$  (note that Ref. [24] fitted fewer parameters). The difference between the two approaches is of order  $\Lambda_{\text{QCD}}^4/(m_c^2 m_b^2)$ . The results are  $|V_{cb}| = (40.8 \pm 0.9) \times 10^{-3}$  [23] and  $|V_{cb}| = (41.1 \pm 1.1) \times 10^{-3}$  [24].

Comparing these shape variables is also the most promising approach to constrain experimentally the accuracy of OPE, including possible quark-hadron duality violation. Quark-hadron duality [25] is the notion that averaged over exclusive channels, hadronic quantities can be computed at the parton level. This is implicitly assumed in the OPE. Duality violations are believed to be small for fully inclusive semileptonic  $B$  decay rates, however, it is hard to quantify how small [26]. One can further test the theory by weighing the lepton spectrum with suitably chosen fractional powers of  $E_\ell$  to reduce the nonperturbative corrections. As shown in Table 1, predictions and data for such “Bauer-Trott moments” [27] are in excellent agreement.

$R_{3a}$	$R_{3b}$	$R_{4a}$	$R_{4b}$	$D_3$	$D_4$
$0.302 \pm 0.003$	$2.261 \pm 0.013$	$2.127 \pm 0.013$	$0.684 \pm 0.002$	$0.520 \pm 0.002$	$0.604 \pm 0.002$
$0.3016 \pm 0.0007$	$2.2621 \pm 0.0031$	$2.1285 \pm 0.0030$	$0.6833 \pm 0.0008$	$0.5193 \pm 0.0008$	$0.6036 \pm 0.0006$

Table 1: Predictions [23] (above) and data [28] (below) for Bauer-Trott moments.

In the future it will be important to determine the  $B \rightarrow D^{(*)}\ell\bar{\nu}$  branching ratios with higher precision, to model independently map out the higher mass charm states in semileptonic  $B$  decay, and to measure the  $B \rightarrow X_s\gamma$  spectrum to as low photon energies as possible. Completing the full two-loop calculation of spectra would also be useful. If these measurements are consistent as they get very precise, the theoretical limitation appears to be around  $\sigma(|V_{cb}|) \approx 3.5 \times 10^{-4}$  and  $\sigma(m_b^{1S}) \approx 35 \text{ MeV}$  [23].

### 3 $B \rightarrow X_u \ell \bar{\nu}$

#### 3.1 $|V_{ub}|$ exclusive: few comments

In  $B$  decays to light mesons there is a much more limited use of heavy quark symmetry than in  $B \rightarrow D^{(*)}$ , since it does not apply for the final state. One can still derive relations between the  $B \rightarrow \rho\ell\bar{\nu}$ ,  $K^*\ell^+\ell^-$ , and  $K^*\gamma$  form factors in the large  $q^2$  region [29]. In the small  $q^2$  region when the energy of the light hadron is large, Soft-Collinear Effective Theory (SCET) [30] can be used to prove a factorization theorem for form factors [31]. The nonfactorizable part satisfies form factor relations [32], while the factorizable part does not. These two terms are of the same order in  $\Lambda_{\text{QCD}}/m_b$ . How they compare numerically, or in the  $m_b \rightarrow \infty$  limit when effects of order  $\alpha_s(m_b)$  and  $\alpha_s(\sqrt{m_b\Lambda_{\text{QCD}}})$  are fully accounted for is an open question.

To determine  $|V_{ub}|$  with sub-10% error, model independent determination of the form factors is needed. This can be achieved in future unquenched lattice QCD calculations, as discussed elsewhere at this conference [33]. Another possibility is to combine heavy quark and chiral symmetries to form “Grinstein-type double ratios” [34], whose deviation from unity is suppressed in both symmetry limits. For example,

$$\frac{f_B}{f_{B_s}} \times \frac{f_{D_s}}{f_D} = 1 + \mathcal{O}\left(\frac{m_s}{m_c} - \frac{m_s}{m_b}, \frac{m_s}{1 \text{ GeV}} \frac{\alpha_s(m_c) - \alpha_s(m_b)}{\pi}\right), \quad (7)$$

and lattice calculations indicate that the deviation from unity is at the few percent level. If three quantities on the left-hand side are measured, the forth is determined with small uncertainty. Similar double ratios can be constructed for the semileptonic decay form factors [35]

$$\frac{f(B \rightarrow \rho \ell \bar{\nu})}{f(B \rightarrow K^* \ell^+ \ell^-)} \times \frac{f(D \rightarrow K^* \ell \bar{\nu})}{f(D \rightarrow \rho \ell \bar{\nu})}, \quad (8)$$

or for appropriately weighted  $q^2$ -spectra in these decays, and may be experimentally accessible soon. Recently the leading power corrections to the HQS relations between the  $B$  and  $D$  decay form factors were analyzed [36]. Replacing  $K^* \ell^+ \ell^-$  by  $K^* \nu \bar{\nu}$  that may be accessible at a super-B-factory would make the deviation of this double ratio from unity very small. With data from hadronic- and super- $B$ -factories the double ratio [37]

$$\frac{\mathcal{B}(B \rightarrow \ell \bar{\nu})}{\mathcal{B}(B_s \rightarrow \ell^+ \ell^-)} \times \frac{\mathcal{B}(D_s \rightarrow \ell \bar{\nu})}{\mathcal{B}(D \rightarrow \ell \bar{\nu})}, \quad (9)$$

could give a determination of  $|V_{ub}|$  with theoretical errors at the few percent level.

### 3.2 $|V_{ub}|$ inclusive: cuts on $B \rightarrow X_u \ell \bar{\nu}$ spectra

If it were not for the huge  $B \rightarrow X_c \ell \bar{\nu}$  background which is  $\sim 50$  times larger than the  $B \rightarrow X_u \ell \bar{\nu}$  signal, measuring  $|V_{ub}|$  would be as “easy” as  $|V_{cb}|$ . The fully inclusive  $B \rightarrow X_u \ell \bar{\nu}$  rate can be calculated in the OPE with small uncertainty [38],

$$|V_{ub}| = (3.04 \pm 0.06_{(\text{pert})} \pm 0.08_{(m_b)}) \times 10^{-3} \left( \frac{\mathcal{B}(B \rightarrow X_u \ell \bar{\nu})}{0.001} \frac{1.6 \text{ ps}}{\tau_B} \right)^{1/2}, \quad (10)$$

where the first error is from the perturbation series and the second is from the  $b$  quark mass,  $m_b^{1S} = 4.73 \pm 0.05 \text{ GeV}$ . If this rate is measured without significant cuts on the phase space, then  $|V_{ub}|$  can be determined with less than 5% theoretical error.

The behavior of the OPE can become significantly worse if kinematic cuts are imposed to distinguish the  $b \rightarrow u$  signal from the  $b \rightarrow c$  background. All such proposed cuts imply (directly or indirectly)  $m_X < m_D$ . Even if many different resonances can be produced in the final state, and therefore the inclusive description is expected to be appropriate, such cuts may still destroy the convergence of the OPE. One may think of the OPE as an expansion of the diagram on the left-hand side of Eq. (5) in powers of  $k$  (which is of order  $\Lambda_{\text{QCD}}$ ), the residual momentum of the  $b$  quark in the  $B$  meson,

$$\frac{1}{(m_b v + k - q)^2} = \frac{1}{(m_b v - q)^2 + 2k \cdot (m_b v - q) + k^2}. \quad (11)$$

In the  $m_b \gg \Lambda_{\text{QCD}}$  limit this expansion converges in most of the phase space. If cuts are applied to the final state phase space, the expansion in  $k$  only converges if the three terms of different orders in  $k$  on the right-hand side exhibit a hierarchy. For  $m_X \ll m_B$  this implies that the range of hadronic final states that are allowed to contribute should satisfy

$$m_X^2 \gg E_X \Lambda_{\text{QCD}} \gg \Lambda_{\text{QCD}}^2. \quad (12)$$

Thus, depending on whether the allowed invariant mass and energy of the hadronic final state (in the  $B$  rest frame) satisfies Eq. (12), there are three qualitatively different regions:

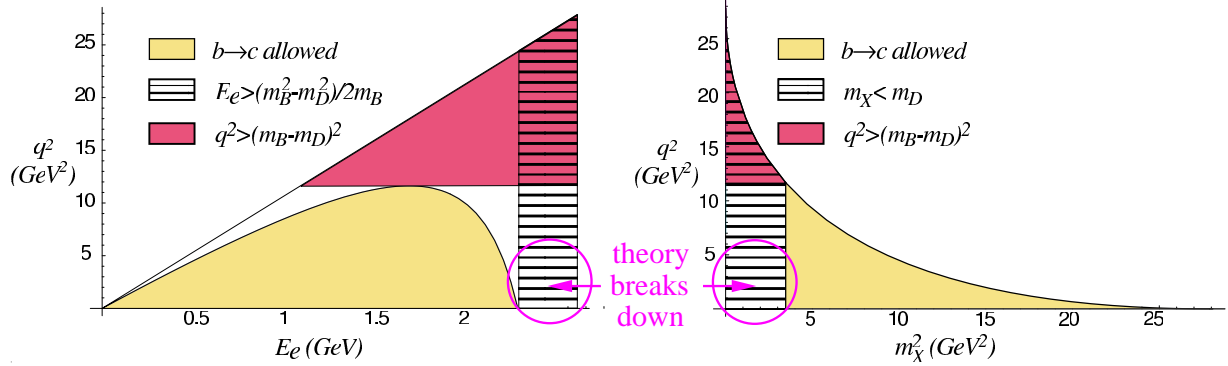


Figure 2: Dalitz plots for  $B \rightarrow X \ell \bar{\nu}$  in terms of  $E_\ell$  and  $q^2$  (left), and  $m_X^2$  and  $q^2$  (right).

- (i)  $m_X^2 \gg E_X \Lambda_{\text{QCD}} \gg \Lambda_{\text{QCD}}^2$ : the OPE converges, and the first few terms are expected to give reliable result. (This is the case for the  $B \rightarrow X_c \ell \bar{\nu}$  rate relevant for measuring  $|V_{cb}|$ .)
- (ii)  $m_X^2 \sim E_X \Lambda_{\text{QCD}} \gg \Lambda_{\text{QCD}}^2$ : an infinite set of equally important terms in the OPE must be resummed. The OPE becomes a twist expansion and nonperturbative input is needed.
- (iii)  $m_X \sim \Lambda_{\text{QCD}}$ : the final state is dominated by resonances, and it is not known how to compute inclusive quantities reliably.

The charm background can be removed by several different kinematic cuts:

1.  $E_\ell > (m_B^2 - m_D^2)/(2m_B)$ : the endpoint region of the charged lepton energy spectrum;
2.  $m_X < m_D$ : the small hadronic invariant mass region [39, 40, 41, 42];
3.  $E_X < m_D$ : the small hadronic energy region [43];
4.  $q^2 \equiv (p_\ell + p_\nu)^2 > (m_B - m_D)^2$ : the large dilepton invariant mass region [44].

These contain roughly 10%, 80%, 30%, and 20% of the rate, respectively. Measuring any other variable than  $E_\ell$  requires the reconstruction of the neutrino momentum, which is challenging experimentally. Combinations of cuts have also been proposed,  $q^2$  with  $m_X$  [45],  $q^2$  with  $E_\ell$  [46], or  $m_X$  with  $E_X$  [47]. These regions of the Dalitz plot are shown in Fig. 2.

The problem is that both phase space regions 1. and 2. belong to the regime (ii), because these cuts allow  $m_X$  up to  $\mathcal{O}(m_D)$  and  $E_X$  up to  $\mathcal{O}(m_B)$ , and numerically  $\Lambda_{\text{QCD}} m_B \sim m_D^2$ . The region  $m_X < m_D$  is better than  $E_\ell > (m_B^2 - m_D^2)/(2m_B)$  inasmuch as the expected rate is larger, and the inclusive description is expected to hold better. But nonperturbative input is needed formally at the  $\mathcal{O}(1)$  level in both cases, which is why the model dependence of the  $|V_{ub}|$  measurement from the  $m_X$  spectrum increases rapidly as the  $m_X$  cut is lowered below  $m_D$  [40].

The spectrum in the large  $E_\ell$  and small  $m_X$  regions are determined by the  $b$  quark light-cone distribution function that describes the Fermi motion of the  $b$  quark inside the  $B$  meson (sometimes called the shape function). Its effect on the spectra are illustrated in Fig. 3, where we also show the  $q^2$  spectrum unaffected by it. This nonperturbative function is universal at leading order in  $\Lambda_{\text{QCD}}/m_b$ , and is related to the  $B \rightarrow X_s \gamma$  photon spectrum [48]. These relations have been extended to the resummed next-to-leading order corrections [49], and to include effects of operators other than  $O_7$  contributing to  $B \rightarrow X_s \gamma$  [50]. Weighted integrals of the  $B \rightarrow X_s \gamma$  photon spectrum are related to the  $B \rightarrow X_u \ell \bar{\nu}$  rate in the large  $E_\ell$  or small  $m_X$  regions. Recently CLEO used the  $B \rightarrow X_s \gamma$  photon spectrum as an input to determine  $|V_{ub}| = (4.08 \pm 0.63) \times 10^{-3}$  [51] from the lepton endpoint region.

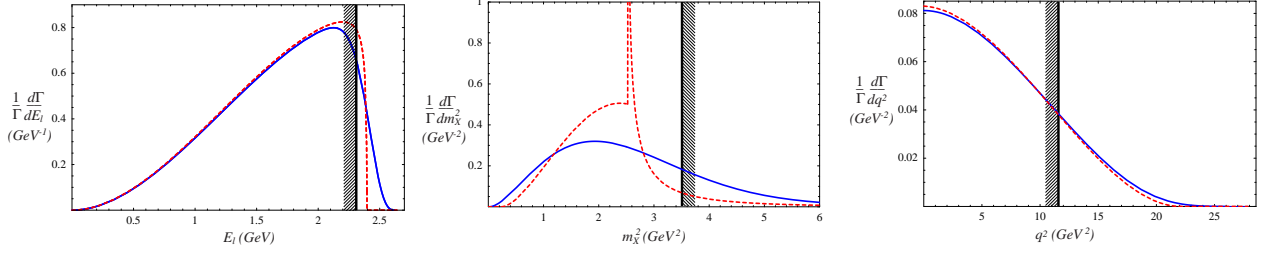


Figure 3:  $E_\ell$  (left),  $m_X^2$  (center), and  $q^2$  (right) spectra. Dashed curves show  $b$  quark decay to order  $\alpha_s$ , solid curves include a Fermi motion model, shading shows the  $b \rightarrow c$  kinematic limit.

The dominant theoretical uncertainty in this determination of  $|V_{ub}|$  is from subleading twist contributions suppressed by  $\Lambda_{\text{QCD}}/m_b$ , which are not related to  $B \rightarrow X_s \gamma$  [52]. The  $B \rightarrow X_u \ell \bar{\nu}$  lepton spectrum, including dimension-5 operators and neglecting perturbative corrections is [17]

$$\frac{d\Gamma}{dy} = \frac{G_F^2 m_b^5 |V_{ub}|^2}{192 \pi^3} \left\{ \left[ y^2(3-2y) + \frac{5\lambda_1}{3m_b^2} y^3 + \frac{\lambda_2}{m_b^2} y^2(6+5y) \right] 2\theta(1-y) - \left[ \frac{\lambda_1}{6m_b^2} + \frac{11\lambda_2}{2m_b^2} \right] 2\delta(1-y) - \frac{\lambda_1}{6m_b^2} 2\delta'(1-y) + \dots \right\}. \quad (13)$$

The behavior near  $y = 1$  is determined by the leading order structure function, which contains the terms  $2[\theta(1-y) - \lambda_1/(6m_b^2) \delta'(1-y) + \dots]$ . The derivative of the same combination occurs in the  $B \rightarrow X_s \gamma$  photon spectrum [53]

$$\frac{d\Gamma}{dx} = \frac{G_F^2 m_b^5 |V_{tb} V_{ts}^*|^2 \alpha C_7^2}{32 \pi^4} \left[ \left( 1 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2} \right) \delta(1-x) - \frac{\lambda_1 + 3\lambda_2}{2m_b^2} \delta'(1-x) - \frac{\lambda_1}{6m_b^2} \delta''(1-x) + \dots \right]. \quad (14)$$

At subleading order, proportional to  $\delta(1-y)$  in Eq. (13) and to  $\delta'(1-x)$  in Eq. (14), the terms involving  $\lambda_2$  differ significantly, with a coefficient  $11/2$  in Eq. (13) and  $3/2$  in Eq. (14). Because of the  $11/2$  factor, the  $\lambda_2 \delta(1-y)$  term is important in the lepton endpoint region [52, 54, 55], giving rise to an order 10% uncertainty. There may also be a sizable uncertainty at sub-subleading order from weak annihilation [56, 52], discussed below.

In contrast to the above, in the  $q^2 > (m_B - m_D)^2$  region the first few terms in the OPE determine the rate with no dependence on the shape function [44]. This is the only differential rate known to order  $\alpha_s^2$  [57]. The  $q^2$  cut implies  $E_X \lesssim m_D$  and  $m_X \lesssim m_D$ , and therefore the  $m_X^2 \gg E_X \Lambda_{\text{QCD}} \gg \Lambda_{\text{QCD}}^2$  inequality is satisfied. This relies, however, on  $m_c \gg \Lambda_{\text{QCD}}$ , and so the OPE is effectively an expansion in  $\Lambda_{\text{QCD}}/m_c$  in this region [58]. The uncertainties come from order  $\Lambda_{\text{QCD}}^3/m_{c,b}^3$  nonperturbative corrections, the  $b$  quark mass, and the perturbation series. Weak annihilation (WA) suppressed by  $\Lambda_{\text{QCD}}^3/m_b^3$  is important, because it enters the rate as  $\delta(q^2 - m_b^2)$  [56]. Its magnitude is hard to estimate, because it is proportional to the difference of two matrix elements, which are equal in the factorization limit. Assuming a 10% violation of factorization, WA could be  $\sim 2\%$  of the  $B \rightarrow X_u \ell \bar{\nu}$  rate, and in turn  $\sim 10\%$  of the rate in the  $q^2 > (m_B - m_D)^2$  region. The uncertainty of this estimate is large. Since this contribution



is also proportional to  $\delta(E_\ell - m_b/2)$ , it is even more important in the lepton endpoint region. Experimentally, WA can be constrained by comparing  $|V_{ub}|$  measured from  $B^0$  and  $B^\pm$  decays, and by comparing the  $D^0$  and  $D_s$  semileptonic widths [56].

Combining the  $q^2$  and  $m_X$  cuts can significantly reduce the theoretical uncertainties [45]. The right plot in Fig. 2 shows that the  $q^2$  cut can be lowered below  $(m_B - m_D)^2$  by imposing an additional cut on  $m_X$ . This raises the scale of the expansion to  $m_b \Lambda_{\text{QCD}}/(m_b^2 - q_{\text{cut}}^2)$ , resulting in a significant decrease of the uncertainties from both the perturbation series and from the nonperturbative corrections. At the same time the uncertainty from the  $b$  quark light-cone distribution function only turns on slowly. The results in Table 2 show that it may be possible to determine  $|V_{ub}|$  with a theoretical error at the  $\sim 5\%$  level using up to  $\sim 45\%$  of the rate. Such accuracy can also be achieved with cuts somewhat removed from the  $b \rightarrow c$  threshold. The experimental status of these measurements was reviewed elsewhere at this conference [3].

Cuts on $q^2$ and $m_X$	Fraction of events	Error of $ V_{ub} $ [ $\sigma(m_b) = 80/30$ MeV]
$(m_B - m_D)^2, m_D$	17%	15%/12%
$6 \text{ GeV}^2, m_D$	46%	8%/5%
$8 \text{ GeV}^2, 1.7 \text{ GeV}$	33%	9%/6%

Table 2:  $|V_{ub}|$  from combined cuts on  $q^2$  and  $m_X$  [45].

In the future there are several ways to reduce the uncertainties: (i) it is important to try to get the experimental cuts as close to the charm threshold as possible; (ii) more precise determinations of  $m_b$  will be useful, since the decay rate is proportional to  $m_b^5$ , and this sensitivity is even stronger in the presence of cuts; (iii) constrain weak annihilation by comparing  $|V_{ub}|$  extracted separately from  $B^\pm$  and  $B^0$  decays, or by comparing the  $D^0$  and  $D_s$  semileptonic widths; (iv) improve measurement of  $B \rightarrow X_s \gamma$  photon spectrum (lower cut) and use it directly in the analysis of the  $E_\ell$  or  $m_X$  spectra instead of via intermediate parameterizations; (v) calculate the full  $\alpha_s^2$  corrections (beyond  $\alpha_s^2 \beta_0$ ), which is only known for the total rate and the  $q^2$  spectrum, but not for others. Clearly, the different  $|V_{ub}|$  determinations have different advantages and different sources of uncertainties. One needs to measure  $|V_{ub}|$  in several ways to gain confidence that the uncertainties are as small as estimated.

## 4 Summary

- $|V_{cb}|$  is known at the  $\sim 4\%$  level, error may soon become half of this. The inclusive measurement can be improved with more precise and consistent data on spectral moments; while the exclusive determination needs  $\mathcal{F}_{(*)}(1)$  from unquenched lattice QCD.
- Model independent determination of  $|V_{ub}|$  with  $\sim 10\%$  error seems possible in the near future. To improve the inclusive determination, neutrino reconstruction with large statistics is crucial; the exclusive needs unquenched lattice form factors or use of double ratios.
- For both  $|V_{cb}|$  and  $|V_{ub}|$ , it is important to pursue both inclusive and exclusive measurements, as they provide powerful crosschecks.

- Progress in SCET in understanding  $B \rightarrow \pi/\rho \ell \bar{\nu}$ ,  $K^* \gamma$ ,  $K^{(*)} \ell^+ \ell^-$  form factor relations in the  $q^2 \ll m_B^2$  region and its experimental tests will affect the sensitivity to new physics in these decays, and may also impact our understanding of charmless nonleptonic decays.
- The theoretical limit in determining  $|V_{cb}|$  and  $|V_{ub}|$  (without lattice QCD) appear to be about 1% and 4%, respectively (achieving these might require a super-B-factory).

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